Available online at www.sciencedirect.com
ScienceDirect
Cement & Concrete Composites 28 (2006) 832–844
Cement & Concrete Composites
www.elsevier.com/locate/cemconcomp

Holistic design of RC beams and slabs strengthened with externally bonded FRP laminates

F. Bencardino [a], V. Colotti [a], G. Spadea [a,*], R.N. Swamy [b]

[a] *Department of Structural Engineering, University of Calabria, Via P. Bucci, Cubo 39/C, 87030 Rende (Cosenza), Italy*
[b] *Department of Mechanical Engineering, University of Sheffield, UK*

Available online 8 September 2006

Abstract

Structural strengthening with externally bonded reinforcement is now recognized as a cost-effective, structurally sound and practically efficient method for rehabilitating deteriorated and damaged reinforced concrete structures. Although a variety of worldwide on-site applications using composite materials have been realized for the rehabilitation and reinforcement of structural elements, the technology is now at a stage where its future development and competitiveness with conventional methods will depend on the definition of valid design guidelines based on sound engineering principles rather than on the availability of new materials or production processes.

The main objective of this paper is to present a general design philosophy for externally plated reinforced concrete beams and slabs, based on a holistic approach, in which appropriate strategies for achieving durable and safe strengthened structures are described.

Essential to the design for safety, durability and ductility is the availability of structural models which are: (i) based on sound engineering principles; (ii) capable of reflecting the physical behaviour of strengthened members; (iii) of general applicability, irrespective of the type of external reinforcement material (steel or fiber-reinforced polymer), and the reinforcement configuration (web or tension plate); (iv) capable of describing all possible failure modes, in order to predict the weakest link chain of resistance of a structural member.

It will be shown, with a series of numerical/experimental comparisons, that such requirements can be conveniently obtained with a unified approach in which materials and structures, calculation and experimental verification, modelling and analysis are integrated.

1. Introduction

Increased global urbanization and evolutionary industrialization that occurred in the world during the latter half of the last century have placed dramatically increased demands on the construction industry in terms of material and energy resources. The present global scenario derived from continued population growth, world-wide urbanization, massive consumption of energy and material resources, wide-spread damage and destruction of infrastructure through ageing, lack of durability, uncontrolled environmental pollution and the resulting global warming with the association of rapid climatic changes, have made the construction industry no longer sustainable [16,29,30]. The main reasons for this state of affairs may be summarized as follows:

- huge amounts of energy and virgin materials are consumed during the production of the construction materials, particularly for portland cement, the production of which—together with the transportation industry—is a major contributor to green-house gases that are implicated in global warming and climate change;
- many concrete structures suffer from lack of durability which has an adverse effect on the resource productivity of the industry. Indeed, when structures deteriorate or fail prematurely, large amounts of energy, material and financial resources are wasted, thus increasing again the world energy demands.

* Corresponding author. Tel.: +39 0984 496919; fax: +39 0984 494045.
E-mail address: g.spadea@unical.it (G. Spadea).

0958-9465/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cemconcomp.2006.07.015

Nomenclature

a	shear span
A_{st}	cross-sectional area of internal stirrups
b	web width of beam
d	effective depth
$d_v = 0.9$ d	effective shear depth
f'_c	cylinder compressive strength of concrete
$f_c = \nu_c f'_c$	effective compressive strength of concrete
h	beam height
M_u	ultimate moment
p_i	equivalent internal stirrup force for unit length
p_e	equivalent external link force for unit length
s	spacing of internal stirrups
U	bond stress resultant at plate/concrete interface
U_y	bond strength
V	shear force
α	ratio of shear span to beam depth
β	ratio of plate length to beam depth
η	longitudinal reinforcement ratio (bars and plate)
θ	angle between diagonal compression stress and beam axis (crack inclination)
ν_c	effectiveness factor for concrete
σ_c	compression stress in concrete
$\tau = V/bd_v$	nominal shear stress
$\phi = U_y/p_i$	ratio of bond strength to internal stirrup strength
$\psi = \psi_e + \psi_i$	total degree of shear reinforcement
ψ_e	degree of external shear reinforcement
ψ_i	degree of internal shear reinforcement
ψ_0	degree of minimum internal shear reinforcement

With the increasing interest of populations and governments in sustainable development, the way to meet the increasing demand for housing and civil infrastructure in a cost-effective and ecological manner is a fundamental challenge facing the construction industry in the new millennium. In particular, sustainable development of concrete and concrete design appear as the most pressing problem of the future, concrete being the primary construction material of the world.

At present, the construction industry poses two major challenges to the concrete technologist and design engineer: first, how to design and build new structures that will have a specified durable service life and, second, how to preserve and maintain the durable service life of existing structures. These challenging goals can be achieved only if the materials manufactured and used, and the structures designed and built or strengthened are cost-effective, give durable service performance over their specified design life and their engineering capabilities are fully utilized and maximized.

To address these issues in order to achieve sustainable development of the construction industry—involving choice of materials, design, construction, maintenance, repair and rehabilitation—the only rational way forward is to develop and adopt a global Holistic Strategy, integrating material characteristics and structural performance. Such a strategy thus advocates a new design philosophy which will involve two distinct but interrelated approaches:

(a) A Material Strategy to develop a high-durability concrete which will ensure sound, reliable and stable behaviour of the material under all possible conditions of load and exposure to which the material in the structure will be subjected to during its service life; for this purpose, we need to adopt the concept of 'Strength through Durability' rather than 'Durability through Strength' [30,31].

(b) A Design Strategy to integrate material properties with structural integrity and ductility which will ensure that the structure will maintain its stability and integrity under the effects of static and dynamic loads, exposure and climatic conditions.

In engineering terms, this implies that the materials we use and the structures we design must be eco-friendly, and have adequate strength and ductility to give durable service life for the period for which they are designed.

The ability for beams and slabs to develop ductile failures and to preserve structural integrity under all load conditions has always been fundamental to structural design. This requirement has now become much more critical in the light of our experience of structural failures all over the world, not only under normal flexural/shear loading regimes but also from dynamic forces arising from floods and hurricanes, for example, due to global warming. These failures clearly show that in reality we need to consider an integrated approach to the design of flexure and shear even in conventional/normal reinforced concrete (RC) beams and slabs.

Most of published data in the available literature show that, when RC beams and slabs are strengthened by externally bonded non-metallic fiber reinforced polymer (FRP) sheets/laminate, brittle failure modes occur. Premature debonding of FRP plate plays an important role in determining such undesirable failure modes. As a result the non-ductile behaviour of the plated beams is not acceptable from an engineering point of view. Methods of suitable plate anchorages have to be carefully designed and set up.

The overall aim of this study is to develop structural models and methods of analysis that will logically reflect the failure processes of plated RC beams, and that can predict not only the failure loads but also the modes of failure. It is also intended that the predictions of the model should be consistently satisfactory, and be applicable to the wide range of test data reported in literature, whatever may be the nature of the material used as plate reinforcement.

2. Beams/slabs strengthened with FRP

The above considerations emphasize that the need to adopt an integrated/holistic design approach becomes much more critical when designing beams strengthened for flexure and shear with externally bonded steel/FRP plates and laminates. A critical review of published literature shows that beams/slabs strengthened with FRP suffer from two major weaknesses, namely, (i) lack of ductility, and (ii) lack of adequate safety in shear. Extensive research studies [8,25,26,28] clearly show that there is a close, synergistic interaction between flexural strengthening and shear strengthening. Strengthening structures separately for flexure and for shear can mask inherent structural weaknesses in ductility and shear, and can lead to premature failures. A Holistic Design approach integrating flexural and shear reinforcement could ensure structural integrity and adequate load capacity under both flexural and shear loading regimes to which a strengthened member could be exposed during its service life. Such an integrated, holistic approach is just as critical to normal RC beams, and a unified global model for both unstrengthened and strengthened beams will clarify the intricate interrelationship between flexural and shear behaviour.

3. Holistic and conceptual design of FRP strengthened beams

3.1. Lack of ductility

When a concrete beam is reinforced with an externally bonded plate, the most important aspect of its behaviour is that composite action should be preserved at all stages of loading right up to failure.

In strengthening for flexure, one normally considers to add plates on the tension face of the beam. Plates cannot be continued over the supports, and the abrupt curtailment of the plate-adhesive system adjacent to a support creates a high concentration of interface shear stress and normal peeling stress in the vicinity of the edge of the plate. The magnitude of these stresses depends on the geometry of the plate reinforcement, the engineering properties of the adhesive and the shear strength of the original concrete beam [19]. The premature debonding and/or anchorage failure can take many forms, and the plate strength also plays an important role in determining the failure mode, and although substantial increases in load capacity can be achieved with FRP plates, the final failure occurs in modes which are not desirable in practice.

Premature and undesirable failures have been reported in RC beams plated with glass, glass–carbon, carbon and aramid fibre reinforced plates, and in the tests reported by [21,25,26], the strain in the compression zone of the concrete never reached the crushing stage for any beam with an FRP plate added to it. The beam tests reported by [21], had been over-designed for shear to avoid brittle shear failures that may occur due to the increased shear load arising from the increased load capacity of composite beams, whilst the beam tests reported by [23], had been under-designed or slightly over-designed for shear, but even then, all the beams with FRP plates demonstrated brittle behaviour, and did not show the yield plateau associated with ductile failures. The behaviour of RC beams reinforced with externally bonded carbon fiber reinforced polymer (CFRP) plates also reveal plate debonding due to peeling off the composite plate resulting in sudden drop in loads and brittle type of failure as showed in the tests of [25,26,33]. Thus, although the CFRP reinforcement is very effective in enhancing both, stiffness and strength, catastrophic failures occurred at the end of the beam load capacities. With increased load capacities of about 40–100%, the failure modes involved debonding failure through the concrete at the internal reinforcement level at the plate end and plate fracture.

The high local interface shear and peeling stresses at the ends of the steel or FRP plates can be effectively countered by the provision of adequately designed bonded anchorage plates [14,26]. Insufficient control of these critical end forces by inadequately proportioned end anchorage plates may bring moderate improvements in structural performance or load capacity, but are unlikely to eliminate premature failures whether the plates are made of steel [14,27] or FRP [17,18,21,22,24,33].

Recent tests reported by [12,26], also confirm that the beams strengthened with only the CFRP laminate bonded on to the tension face without any external anchorages, carried higher loads at failure but they failed suddenly in a brittle manner, with the load capacity dropping substantially at the instant of failure, which occurred by explosive debonding of the CFRP plate. The strengthened beams with the end anchorages and other additional bonded anchorages produced less brittle failures and carried higher failure load than the strengthened beams without external anchorages.

In order to clarify this issue, a multiphase experimental/analytical research has been carried out by the authors. In Figs. 1 and 2 the most relevant geometrical details of the tested beams are given.

It is clear from these results that strengthening RC beams with CFRP plates without consideration of the very complex high stresses at the ends of the plate, and consequent bond slip between the plate and the CFRP laminate can lead to unacceptable failure modes in practice. A critical study of the test results and the failure modes of the beams show that one of the roles of the anchorages is to transform a brittle type of failure into an almost ductile failure.

The end and intermediate anchorages control the critical normal/shear and peeling stresses which lead to bond slip, debonding of the bonded element, and premature failure help the bonded reinforcement to carry further loads.

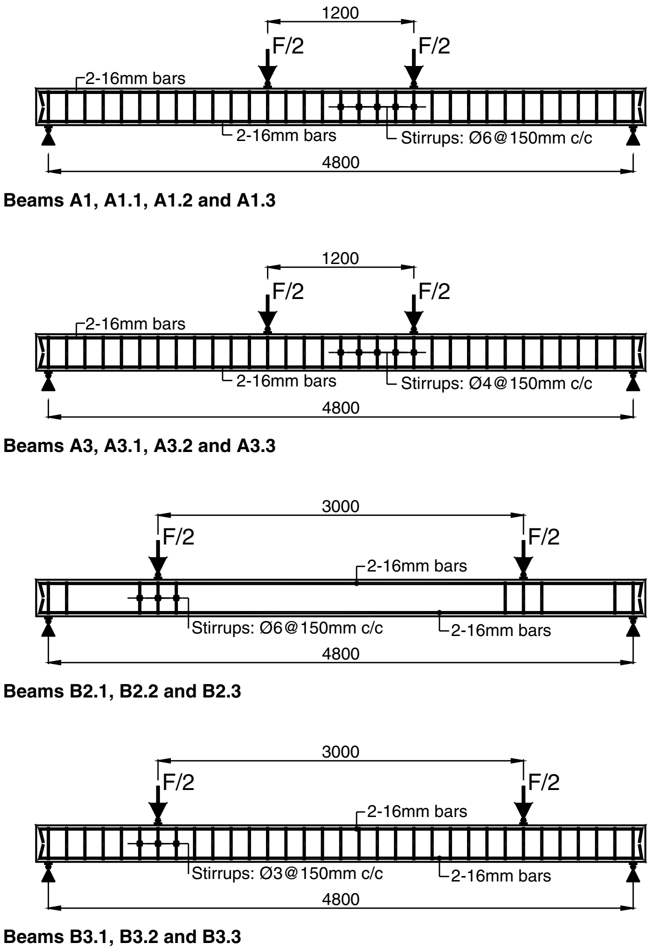

Fig. 1. Internal reinforcement details and load positions.

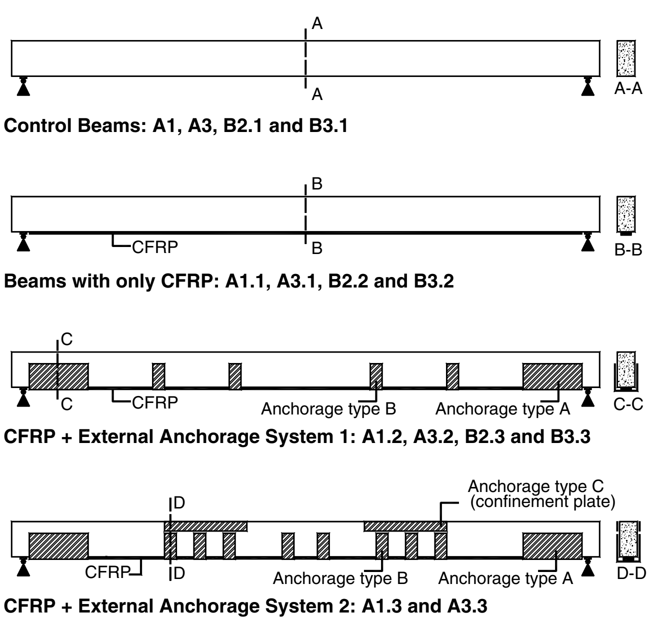

Fig. 2. External reinforcement details.

It is also important to note that the provisions of these bonded anchorages prevented the rupture of the CFRP laminate, a phenomenon observed in tests without bonded anchorages whilst the type C anchorage helped to prevent/control premature failure of the compression zone.

To give some idea of the interrelationship between load-carrying capacity and ductility of beams strengthened with CFRP compared to those of their original unplated beams, the increase in the load-carrying capacity and decrease in deflection ductility over those of the control beams is shown in Fig. 3. It can be readily seen that while CFRP materials as bonded external reinforcement can achieve significant increases in strength, these are obtained at the expense of ductility. Careful design of the end and other anchorages can make major contributions in regaining a substantial part of the lost ductility property, but is unable to restore the strengthened beams to the original levels of ductility.

3.2. Lack of safety in shear

A clear understanding of the mechanisms involved in the role of shear in RC beams strengthened by external plate bonding requires a lot of test data which would help to allow identify the effects of various parameters related to shear behaviour. There is still a lack of information on the effectiveness of bonded plates in resisting shear forces and their contribution to shear strength.

Some test data are available on the behaviour of FRP plated beams in shear [1,12,27,28,34] and these results show that plates bonded to the tension face alone or only to the webs of the beams cannot enhance the shear capacity of the beams with no or inadequate internal shear links to the values required to produce ductile flexural failures.

All the available test data show that efficient anchorage of the ends of the tension plates and protection of the concrete shear zone at the ends of the beam are both critical to shear but, in addition, both the geometry of the beam and the presence or absence of internal stirrups are also very influential in determining both the ultimate shear capacity and failure mode of the plated beams [5,7].

The problem of shear is a complex phenomenon and both the shear strength and mode of failure are influenced by many factors. It is, of course, possible to bond plates over the full depth of the webs and over the full length of the beams or restricted to the shear spans only, and often this can enhance the load capacity of the beams equivalent to that of flexural failure. However, continuous web plates in the shear spans or over the full length of the beams are likely to create undesirable secondary failures [1,15] and, in any case, continuous web plates do not appear to be the most attractive method of enhancing shear strength. Anchoring the compression zone above the intermittent strips (see type C anchorage in Fig. 2) is a useful way of enhancing the shear capacity and develop flexural failure, but even then undesirable shear cracking cannot be avoided totally.

Furthermore, beams strengthened for flexure need to be checked to ensure that a brittle failure associated with

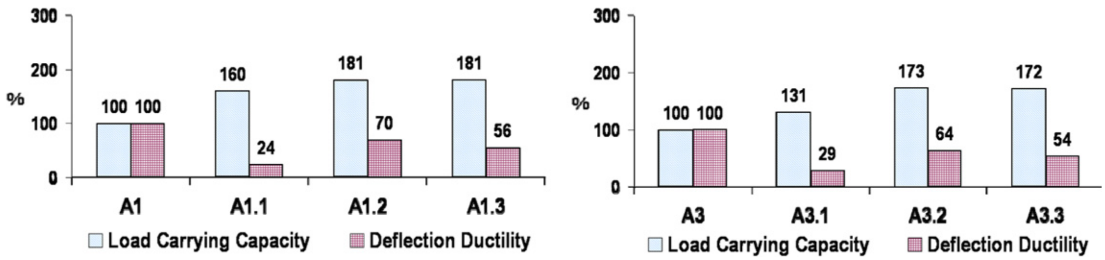

Fig. 3. Increase of load-carrying capacity and decrease of deflection ductility.

shear forces does not occur in practice. It is therefore important to understand the effects of a predominant shear loading regime on the strength, stiffness, and ductility of a RC beam strengthened in flexure by external plate bonding at the tension face.

The test results reported by [8] on RC beams strengthened on the tension face but exposed to a predominant shear loading regime, show two significant implications. Firstly, strengthening an RC beam with a CFRP plate bonded to the tension face alone without considering the end anchorage stresses and the bond slip between the sheet and the concrete substrate can lead to substantial losses in structural performance. Secondly, a judicious combination of the end anchorages together with the tension CFRP plate can substantially overcome this loss, and enable the strengthened beams to increase the structural performance of the original unplated control beam and/or simply plated beam. The overall implication of the obtained data is that when we design bonded plates to strengthen and rehabilitate existing beams, adequate anchorages at the plate ends are essential in order to guarantee the best structural performance characteristics of the strengthened RC beam. The additional anchorage system modifies the failure mode of the strengthened RC beam under predominant shear forces from a brittle failure, without any significant increase of load capacity, to an almost ductile failure with a substantial increase of load-carrying capacity with yielding of the tension steel.

3.3. What do all these tests tell us?

Strengthening of beams should not be seen as strengthening for flexure or strengthening for shear but as an unified approach of strengthening for flexure and shear. To preserve structural integrity and structural safety, a synergistic interaction between tensile strengthening and shear links is essential. In fact, recent studies [8,26] emphasize this interaction and imply that external shear links can act compositely with external longitudinal reinforcement, and that such shear link–longitudinal plate interaction is synergistic with significant structural benefits. In practice, the role of shear links is two-fold: they not only act as a confinement tool to improve the effectiveness of the flexural sheets before the eventual flexure or shear failure, but also as externally bonded shear reinforcement to improve the shear resistance of the beam. A direct result of these synergistic interactions is that shear links are very effective in controlling the tensile splitting at the level of the main reinforcement and the peeling in the vicinity of the cut-off ends of the tension plate, which are both characteristic of premature and brittle failures. At the same time, the shear links are equally very effective in controlling the growth of shear cracks and thereby make the mechanisms which govern the shear resistance of the beam more active. The benefits of this double fundamental role of shear links lead to better structural performance in terms not only of ultimate failure loads but also in terms of deformational behaviour. The actual possibility to transform the mode of failure from a sudden and catastrophic mechanism into a more gradual and ductile shear or flexure failure depends obviously on many factors, but the presence of both longitudinal and transverse reinforcement is indispensable for the structural member to be able to mobilize more efficiently the different mechanisms to resist loading. More importantly, the shear links are able to guarantee the structural integrity of the member and are able to prevent or delay premature and brittle failure modes due to plate debonding mechanisms.

These considerations and recommendations are supported from the experimental data shown in Table 1, which refer to the tested beams shown in Figs. 1 and 2. In this table are shown the most relevant results obtained from experimental tests carried out on beams subjected to four point bending with different shear span-to-effective depth ratio (a/d).

With reference to the beams tested with $a/d = 6.9$, the results show that all the tested beams, unplated and plated, failed after yielding of the tension steel. Specifically, the unplated control beams failed in flexure at an ultimate load of 54.0–57.2 kN, after extensive yielding of the tension steel followed by crushing of the concrete in the compression zone. The maximum concrete strain in the compression zone at mid-span, just prior to failure, was over 5000 μm/m. The simply plated beams, strengthened with only the CFRP laminate bonded on the tension face without any external anchorages, carried higher loads at failure, but both beams failed suddenly in a brittle manner, with the load capacity dropping substantially at the failure stage, which occurred by explosive debonding of the CFRP plate.

Compared to the control beam the load at ultimate increased from +30.8% to +60.7%, whereas, the strain in the compression concrete measured just prior to failure

Table 1
Loads and strains at critical stages

Tested beams	Beams labels	Tension steel yielding load (kN)	Failure load (kN)	Max concrete strain at failure (μm/m)	CFRP strain at failure (μm/m)
a/d = 6.9					
Control beams	A1	45.2	54.0	5000	–
	A3	40.2	57.2	6300	–
Beams with only CFRP	A1.1	55.3	86.8	2100	7100
	A3.1	54.9	74.8	2700	6100
CFRP + external anchorage system 1	A1.2	59.8	98.0	3700	9600
	A3.2	55.1	98.8	4300	10,200
CFRP + external anchorage system 2	A1.3	55.2	96.7	4400	10,500
	A3.3	55.0	98.3	4500	12,000
a/d = 3.4					
Control beams	B2.1	No yield	82.5	871	–
	B3.1	No yield	80.1	893	–
Beams with only CFRP	B2.2	No yield	82.1	1063	1981
	B3.2	No yield	82.7	1020	1922
CFRP + external anchorage system 1	B2.3	134.9	206.3	2706	8669
	B3.3	120.2	203.5	2735	8474

was 2100–2700 μm/m, which is 43–45% of the strain at ultimate measured in the control beams.

The strengthened beams with the end anchorages and other additional bonded anchorages produced more ductile failures and carried still higher failure loads than the strengthened beams without external anchorages. These beams developed compressive strains at failure ranging from 3700 to 4500 μm/m, whereas the CFRP laminates reached strains varying from 9600 to 12,000 μm/m, which on average is 60.2% higher than the strain measured on CFRP in the simply plated beams.

The brittle mode of failure due to debonding is reflected in the low strains at failure in the CFRP plate and in the compression concrete of the beams with only CFRP, as shown in Table 1. The end and intermediate anchorages help the bonded reinforcement to carry further loads, enhance the concrete strain in compression zone, and improve the performance characteristics of the strengthened beams. The almost ductile failure mode is reflected in the higher strains at failure measured in the CFRP plate and in the compression concrete of the plated beams with additional external links.

The control beams tested with $a/d = 3.4$ failed typically in shear and in a brittle manner with a diagonal shear crack starting from near one of the supports. Both the beams failed at nearly the same ultimate loads of 80.1–82.5 kN, with again very similar values of concrete strain at the extreme compression face of 870–895 μm/m. However, the strain in the tension steel in these beams was less than the yield strain, as would be expected. The beams strengthened with only a bonded CFRP lamina at the tension face behaved very similarly, not only to each other, but also to the control beams. Both the beams failed in shear at almost the same ultimate load (82.1–82.7 kN), and had almost the same concrete compression strain at mid-span at failure (1020–1065 μm/m). Further, the maximum tensile strain in the CFRP in the two beams was also of the same order of magnitude of 1920–1980 μm/m, which is only about 14% of its ultimate failure strain. These results show that bonding a CFRP lamina at the tension face of an RC beam inherently weak in shear, will be unable to modify the structural response of the beam or to increase the beam strength when it is subjected to load positions with dominant shear behaviour. The beams with one bonded CFRP at the tension face but with external anchorages showed vastly improved structural behaviour compared to the similar beams with only CFRP. They also showed almost identical structural behaviour. Both the beams carried much higher loads and failed in a much less brittle manner, allowing steel yielding to take place, compared to the failure modes of control and simply plated beams, where the steel did not reach the yield point. In both these beams, the tension steel yielded before failure, while the strain in the compression concrete reached values of 2705–2735 μm/m, almost near that normally observed in flexure failure. The external anchorages also helped the bonded CFRP lamina to be structurally more effective. In fact, their tensile strains reached values of 8475–8670 μm/m, corresponding to more than four times the strain measured at failure on the CFRP laminate in the simply plated beams.

The increased load-carrying capacities of the beams with the external U-shaped anchorages, 2.5 times higher, can be attributed to the combined effects of the anchorages as externally bonded shear reinforcement for the concrete beam, and their effect as anchorage for the external flexural reinforcement. Obviously, the anchorages allow the flexural strengthening laminate to be effective.

3.4. Performance factor

Ductility is of fundamental importance in the design of RC structures using conventional materials such as concrete and steel. However, when structural members are strengthened with externally bonded FRP laminates the conventional definitions of ductility alone, defined in terms of deflection, curvature or energy absorption capacity, without considering the increase of strength, become inadequate in defining the structural effectiveness of the external strengthening because of the brittle behaviour of FRP materials, their resultant effect on the performance of the strengthened beam and/or the possible occurrence of premature debonding failure mode. Therefore, such beams when strengthened without the provision of adequate external anchorages will fail in a brittle manner with abrupt detachment of the laminate and a sudden loss of their load-carrying capacity.

Conventional definition of ductility defined as the ratio of a terminal plastic/terminal elastic quantity (such as curvature, deflection, energy, etc.) is not appropriate for externally strengthened beams with FRP materials because only a portion of their deformations at ultimate are plastic. Indeed even the portion corresponding to the plastic energy dissipated by the yielding of the steel may not be recoverable if secondary debonding failures occur before all the energy is dissipated.

Consequently, it is worthwhile to evaluate the overall structural performance of such strengthened composite beams, and the actual efficiency of the external anchorage system. Unlike the concept of toughness as applied to single materials, the performance factor (PF) incorporates both the deformability and strength behaviour of composite beams and the effect of other parameters which influence structural design.

Because of the nature of the stress–strain profile of FRP materials, it is rational, from an engineering point of view, to relate the deformability and strength factors to the two critical stages in the design process and behaviour characteristics of structural members, namely, the ultimate limit state and the serviceability limit state.

In general terms, the serviceability limit state can be satisfactorily defined for most structural systems as the stage when the compressive strain in the concrete begins to have a non-linear behaviour at a strain of 0.001.

These two factors can then be defined as

$$\left.\begin{array}{l}\text{Deformability factor, } \mathrm{DF} = \dfrac{\text{deflection at ultimate limit state}}{\text{deflection at } \varepsilon_c = 0.001} \\ \text{Strength factor, } \mathrm{SF} = \dfrac{\text{load at ultimate limit state}}{\text{load at } \varepsilon_c = 0.001}\end{array}\right\}, \tag{1}$$

where, ε_c is the concrete compressive strain.

The two factors—the deformability factor (DF) and the strength factor (SF)—thus recognize the two critical factors of design, namely, strength and deformability or ductility. The overall structural performance of the strengthened composite beam can then be evaluated by a single criterion, namely, the performance factor, by integrating the strength factor and the deformability factor, defined as

$$\mathrm{PF} = \mathrm{DF} \times \mathrm{SF}. \tag{2}$$

This PF can be seen as a global factor, integrating strength and deformability. For the designer both factors are important and for the best structural performance, strength and deformability need to be optimized. On the other hand, the concept of PF also gives to the designer the opportunity to balance the strength and deformability requirements of a particular application, as in seismic design and emphasize thus the need to consider both strength and deformability when selecting materials and design parameters.

The effectiveness and reliability of this parameter has been examined by the authors with reference to a series of fourteen RC beams which were tested to evaluate the structural performance of beams strengthened with and without externally bonded CFRP laminates, and with different types of internal reinforcement and external anchorage systems under different loading conditions. The structural behaviour of eight beams tested with a predominant flexure loading regime [25] and other six beams tested with predominant shear loading regime [8], were evaluated by the performance factor.

The values of PF of the all mentioned beams, grouped with reference to the external reinforcement details, are given in Tables 2 and 3, respectively. The most important implication of the results shown in Table 2 is that beams strengthened without any external anchorages, will have substantial reductions in their performance factors. For example, beams A1.1 and A3.1, strengthened with a CFRP sheet without any external anchorages, had a PF of less than a fifth of that of the corresponding control beams A1 and A3. These results thus show that external anchorages are an essential component of design for plate-bonded composite beams.

Table 2
Various factors for beams under flexure loading regime

Flexure loading regime: $a/d = 6.9$		Factors: deformability, strength, performance		
Tested beams	BEAM	DF	SF	PF
Control beams	A1	11.88	2.16	25.66
	A3	11.23	2.20	24.71
Beams with only	A1.1	2.55	1.85	4.72
CFRP	A3.1	2.66	1.78	4.73
CFRP + external	A3.2	6.01	2.35	14.12
anchorage system 1	A1.2	5.76	2.09	12.04
CFRP + external	A1.3	5.28	2.49	13.15
anchorage system 2	A3.3	7.08	3.07	21.73

Table 3
Various factors for beams under shear loading regime

Shear loading regime: $a/d = 3.4$		Factors: deformability, strength, performance		
Tested beams	BEAM	DF	SF	PF
Control beams	B2.1	<1	<1	<1
	B3.1	<1	<1	<1
Beams with only	B2.2	1.08	1.06	1.14
CFRP	B3.2	1.02	1.02	1.04
CFRP + external	B2.3	2.10	3.40	7.14
anchorage system 1	B3.3	3.80	2.36	8.97

Considering the results shown in Table 2, it is clear that both the internal reinforcement details and the type of external anchorage system have a profound influence on the most effective method of enhancing the structural performance of a strengthened beam. In other words, for a given internal reinforcement system, there is a possible optimum external anchorage system which will maximize the overall structural performance Thus, the external anchorage systems 1 and 2 were more effective in optimizing the performance characteristics of the CFRP strengthened beams of series A1 and A3. In fact, beams A1.2, A3.2, A1.3 and A3.3 were able to achieve some 50–90% of the performance characteristics of their corresponding control beams. The results in Table 2 confirm that strengthening existing RC beams by the plate bonding technique would be strongly influenced by the design details of the structural element to be strengthened; and this relative structural performance is clearly reflected in the PF. Both the internal reinforcement details of the beams to be strengthened and the type of anchorage system influence the overall PF. Referring to Table 2, the beams A1 and A3 have almost the same PF; the beams A1.1 and A3.1 have almost the same PF; the beam A1.3 has a PF higher than the beam A1.2; the beam A3.3 has a PF higher than beam A3.2. These results imply that with the complex failure mechanism of a strengthened beam, the type of anchorage has a profound influence in failure modes of the strengthened beam.

External anchorages at the ends (Anchorage type A) of the bonded plate are essential in this design process whilst the arrangement and disposition of the intermediate anchorages (Anchorage type B) are influenced by the relative amount of internal reinforcement. A properly selected combination of the end and intermediate anchorages can significantly improve performance characteristics, both in terms of the ultimate strength and ductility/energy absorption properties of the strengthened composite beam. The PF reflects the overall structural behaviour of the control unplated beams, and that of the CFRP plated beams. In addition, this parameter is capable of identifying the role and effectiveness of external anchorages in optimizing the load–behaviour characteristics of the CFRP plated beams.

The data in Table 3 show that the capacity for strength and deformability of the control beams (B2.1 and B3.1) and of the simply strengthened beams (B2.2 and B3.2) were drastically reduced, and that their overall performance are also very much adversely affected. On the other hand, the factors related to beams with an external anchorage system 1 (B2.3 and B3.3) show that a well positioned external anchorage system can significantly improve performance characteristics, both in terms of strength and deformability.

It can be readily seen that these results have two significant implications. Firstly, strengthening an RC beam with a CFRP plate bonded to the tension face alone without considering the end anchorage stresses and the bond slip between the sheet and the concrete substrate can lead to substantial losses in structural performance as shown by beams B2.2 and B3.2. The value of the various factors of these beams is about one. Secondly, a judicious combination of the end anchorages together with the tension CFRP plate can substantially overcome this loss, and enable the strengthened beams to increase, on average, about eight times the PF of the original unplated control beam and/or simply plated beam. The overall implication of the data in Tables 2 and 3 is that when we design bonded plates to strengthen and rehabilitate existing beams, adequate anchorages at the plate ends are essential in order to guarantee the best structural performance characteristics of the strengthened RC beam. Moreover, there is a distinct loss in structural performance characteristics of the simply strengthened beams, when compared to the strengthened beams with external end anchorages.

Bonding a CFRP lamina to the tension face of RC beams, weak in shear, is not an adequate structural solution to increase their load bearing capacity or to change their mode of failure. The ultimate failure load of the strengthened RC beam is almost the same as the failure load of the unstrengthened RC beam when the shear loading regime is predominant, and the mode of failure remains extremely brittle.

The anchorage system enables, before failure, the tension reinforcing bar to yield, the CFRP lamina to reach a high proportion of its failure strain, and allows the compression concrete to reach a high strain capacity, almost equal to the failure strain in flexure. Therefore, the anchorage system enhances the strength and deformability properties of the CFRP plated beam, as reflected by the global PF, and modifies the failure mode of the strengthened RC beam under predominant shear forces from a brittle failure, without increase of load capacity, to an almost ductile failure with a substantial increase of load-carrying capacity with yielding of the tension steel.

These results show that the use of the PF can realistically represent the overall structural behaviour of the strengthened composite beams both in terms of strength enhancement, ductility and deformability behaviour. It is also shown that the reinforcement details of the beams to be strengthened have a strong influence on the effectiveness (arrangement and position details) of the external anchorages designed to optimize the structural behaviour of the strengthened composite beams.

4. Holistic design and modeling of externally strengthened beams/slabs

Within the past ten years, many analytical and experimental investigations have been carried out in order to develop design/analysis procedures and code specifications in the area of repair and strengthening of RC structures.

In analyzing strengthened RC beams/slabs it is common practice to carry out distinctly separate analyses for flexure and for shear. Shear strengthening is required when a structural member is found deficient in shear, or when its shear capacity falls below its flexural capacity after flexural strengthening. These steps comprise a sequence of interdependent analyses where one analysis, as currently performed, is independent from the other. Although this current approach reflects a compromise aimed at bridging the complexities of the phenomena involved together with insufficient consolidated input data available, there is no doubt that a need for a coupled approach that considers flexure and shear simultaneously, would be able to account for the close synergistic interaction discussed above.

For RC members strengthened in flexure and/or shear, several studies [2,21] have shown that the flexural mechanism can reasonably be predicted by using the same analytical models applicable to conventional RC beams and based on the simple flexure theory. On the contrary, when the shear and/or debonding mechanisms govern the response of a strengthened RC member, the problem of a correct prediction of ultimate strength is more complicated, due to the complexities involved with these mechanisms.

Many studies have been carried out using the finite element method to predict the structural behaviour of plated RC beams [3,9,32,35,36], incorporating sometimes the debonding effects. However, because the plate debonding phenomenon is very complex, and the factors involved are numerous and interdependent, proper modelling of a composite structure such as a plated RC beam requires significant effort even within the regime of linear elastic analysis [32]. The key factors affecting the reliability of a numerical analysis in simulating debonding failures in plated RC beams are the proper simulation of crack propagation and the load transfer at the plate–concrete interface. The current approaches representing cracking in concrete structures, based on the discrete crack approach and the smeared crack approach, present some limitations and deficiencies such that the output from these numerical analyses are not always consistent or reliable. Moreover, to account for the possibility of debonding failure in numerical modelling, it is necessary to model the bond at the plate–concrete interface. For this purpose, the incorporation in the finite element modelling of interface elements, such as link or contact elements, represent a suitable solution. The accuracy of the predictions of the response of strengthened RC members, however, depends on the bond stress–slip relationship that was chosen for the analyses. The difficulties related to the lack of completely reliable basic models, mainly due to the many interrelated factors which influence the bond–slip relationship, together with the complexities involved in detailed solutions greatly limit the use of finite element method in the design and analysis of strengthened RC beams. Nevertheless, when the problem of calibration of the key factors which affects the behaviour of the different materials bonded is overcome, promising results have been published in literature [4,6,13].

5. The strut-and-tie model in the context of the holistic design approach

The uncertainties and complexities involved in the formulation of a detailed finite element model for strengthened RC beams necessitate the development of other simplified analytical models capable of predicting a specific overall behaviour of whole structural elements with reasonable accuracy. Recent years have seen the emergence of the strut-and-tie models as a powerful general approach for the rational and consistent design of structural concrete members. The idea of the strut-and-tie model comes from the truss analogy method introduced originally by Ritter and Mörsch about one hundred years ago for the shear design of RC beams. With the increasing availability of experimental results and the development of limit analysis in the plasticity theory, the truss analogy method has been greatly improved and validated in the last few decades. Being based on careful observations of the behaviour of tested structures as well as on a more fundamental understanding of basic principles, models derived from truss analogy provide powerful tools for the structural dimensioning and detailing of a wide range of applications. Current truss models, however, do not realistically model load transfer by bond and, thus, lack the capability of explaining failure modes influenced by bond–slip failure, as in the case of strengthened RC beams. To overcome this deficiency, the plasticity truss model developed primarily by [20], for ordinary RC beams has been generalized to extend its applicability for strengthened RC beams [10,11]. As discussed in the following, the strut-and-tie approach provides all the essential requirements of holistic design for structural strengthening.

The essential requirements of holistic design for structural strengthening should be based on sound/rational engineering principles and it should consider all possible failure modes and especially incorporate the load transfer mechanism by bond to reflect the debonding phenomenon which has a dominant influence on the failure processes of plated beams. Holistic design structural approaches will give designers a physical understanding as to how the strengthened structure will fail in real life loading/environmental conditions.

The strut-and-tie model satisfies most of the essential requirements of holistic design. In fact, it is based on sound engineering principles and the basic assumptions do not rely on any empirical equations except on bond slip—this is because of our lack of insight into bond mechanism both in normal RC beams and strengthened beams—particularly when the bond is epoxy bonded systems. However, conventional truss models ignore any load transfer by bond and are, therefore, not applicable to plated beams. The truss model proposed here incorporates this load transfer by bond, thereby providing an appropriate mechanism to reflect the various modes of failure observed in tests with plated beams, including those influenced by bond slip. In the following a unified, global model is described which represents a generalized formulation of the basic strut-and-tie philosophy, valid for RC beams strengthened both for flexure and/or shear with externally bonded reinforcement made of steel or FRP materials.

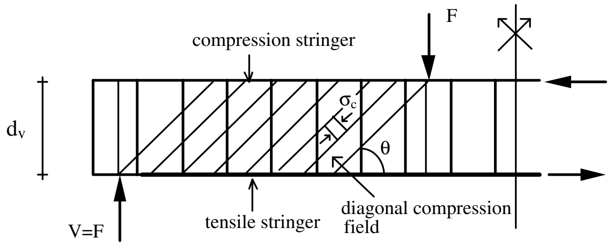

Fig. 4. Truss analogy model for strengthened RC beams.

The load-carrying capacity of a RC beam with externally bonded reinforcement in the form of a tension plate and web laminate or vertical strips can be determined with reference to a scheme based on the truss analogy model. In this approach, a beam is seen as a plane truss with an upper and a lower chord (compression concrete and tensile reinforcement, respectively), and a web element (diagonal concrete struts and transverse stirrups) as shown in Fig. 4. To extend the applicability of the truss analogy concept for strengthened RC beams, in the development of the model, the following assumptions are made:

- perfectly plastic behaviour is assumed for the materials and for the bond at the plate–concrete interface. In particular, it is assumed that the failure of the external reinforcement occurs after yielding of the internal steel stirrups, according to the current design concept of beams for which the steel yielding/concrete crushing occur before FRP fracture or debonding failure;
- the concrete strut in the web is subject to a uniaxial compressive stress state σ_c inclined at an angle θ with respect to the beam axis (diagonal compression field);
- the strength of the concrete web at crushing is $f_c = v_c f'_c$, where f'_c is the cylinder strength and v_c is a suitable web effectiveness factor, introduced to take into account the limited ductility of concrete material;
- the external tension plate is treated like a conventional reinforcement. Perfect bonding between plate and concrete is assumed;
- to take account of the force transfer mechanism for bonding, and the associated debonding mechanism, the bonding force U is considered to represent the plane stress flow at the plate–concrete interface. At the ultimate stage, the distribution of the bonding forces between plate and concrete is assumed to be uniform;
- when the external plate bond mechanism is the failure criterion, the contribution of the internal longitudinal bars at the ultimate state is neglected. The dowel action of the reinforcement and the aggregate interlock effects are also neglected;
- the shear force V provokes an average shear stress τ along the web element, given by $\tau = V/bd_v$, where $b =$ thickness of the web and $d_v =$ depth of the web, equal to the distance between the upper and bottom stringers (Fig. 4);
- the internal shear reinforcement of the beam, provided by vertical stirrups, arranged at a uniform spacing of s, is subject to an equivalent unit length force p_i whose value at yielding is given by $p_i = A_{st} f_{ty}/s$, where A_{st} and f_{ty} are the cross-sectional area and the tensile yield strength of the stirrups, respectively;
- the external shear reinforcement, in the form of continuously bonded plate/laminate or discontinuously bonded vertical strips (Fig. 5), is treated like conventional internal stirrups, in terms of distributed forces for unit length p_e. Its quantitative evaluation at failure depends on the failure mode, that is, bond slip or tensile fracture;
- the contributions to the shear resistance due to the internal and external shear reinforcement are assumed to be additive, and given by the sum of the respective degree of shear reinforcement, $\psi = \psi_i + \psi_e$, where $\psi_i = p_i/bf_c$ refers to the internal stirrups and $\psi_e = p_e/bf_c$ refers to the externally bonded links.

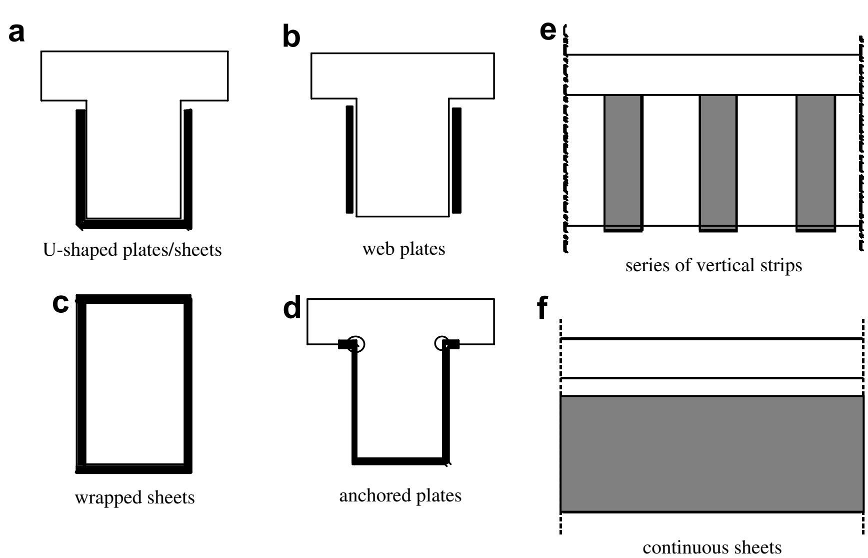

Fig. 5. Shear strengthening configurations.

According to the lower bound theorem of the theory of plasticity and the pioneering work of [20], a series of non-dimensional relations for the shear capacity of RC beams strengthened to flexure and/or shear, corresponding to different failure modes, has been derived [10,11]. In practice, the ultimate capacity of the strengthened beam can be determined as follows. For the sake of brevity all the symbols are listed in the nomenclature. Further details and proper significance are given in the referred papers.

5.1. Shear failure mode (web crushing)

The concrete web stress reaches the effective compressive strength and the internal/external stirrups reach (or not) their limit stresses, depending on the values of the shear reinforcement degree ψ

$$\frac{\tau}{f_c} = \sqrt{\psi(1-\psi)} \quad \text{for } \psi_0 \leqslant \psi \leqslant 0.5. \tag{3}$$

5.2. Tension/concrete crushing failure mode

The internal/external stirrups and the longitudinal internal/external reinforcement reach the yield/fracture stress:

$$\frac{\tau}{f_c} = \psi\left[\sqrt{\frac{2\eta}{\psi} + \alpha^2} - \alpha\right] \quad \text{for } \psi > \psi_0. \tag{4}$$

5.3. Flexural failure mode

The longitudinal internal/external reinforcement reaches the yield/fracture stress and/or the concrete crushing in the compression stringer:

$$\frac{\tau}{f_c} = \frac{M_u}{abd_v f_c}. \tag{5}$$

5.4. Plate debonding failure mode

The internal stirrups reach the yield stress while the bonding forces between external tensile plate and concrete reach the bond strength:

$$\frac{\tau}{f_c} = \psi_i\left[\phi + \alpha - \sqrt{(\phi+\alpha)^2 - 2\phi\beta}\right] \quad \text{for } \psi_i > \psi_0. \tag{6}$$

This relation is valid only for members strengthened to flexure, without external shear link. Furthermore, because the mechanism associated with the bond failure involves both the slipping of the external tensile plate and the yielding of the stirrups, it is restricted to elements with a minimum internal shear reinforcement, i.e., for $\psi_i > \psi_0$. It is important to emphasize that in the philosophy of the holistic design, the strengthening scheme made of only external tensile plate without the support of external shear links should be avoided.

The actual load-carrying capacity of a strengthened beam is then determined by the minimum value obtained for the different failure mechanisms considered in the analysis. More details of the model can be found in [10,11].

6. Validation of the model

The proposed structural model, based on the modified strut and tie approach, has been applied to the results of about 200 RC beam tests reported in literature, for which the experimental data are available. The beams differ in terms of geometry, concrete strength, and amount and strength of internal steel reinforcement (both longitudinal and transverse), with a wide range of geometrical and mechanical characteristics. The results of the experimental validation are discussed below.

6.1. RC beams strengthened for flexure

Fig. 6 presents a graphical comparison between the experimental failure loads and the analytical results based on the proposed structural model. The actual load-carrying capacity of each beam, V_{mod}, is the minimum of the four failure loads corresponding to each failure mechanism. The results in Fig. 6 show very good agreement between the experimental and analytical ultimate loads. The mean value of V_{exp}/V_{mod} ratios is 1.04, while the coefficient of variation (COV) is 16.3%. The results obtained also show that about 60% of the failure modes observed experimentally are the same as those predicted by the analysis. However, in the authors opinion, in many cases the differences in classification of the failure mode between the model and that observed experimentally, is not a weakness of the model; rather, the misclassification can be attributed possibly to the inadequate and improper understanding of the real causes of failure and the consequent misclassification of the failure mode on the part of the authors based on visual observations without regard to the actual state of stress in the beam at the instant of failure. The quality of

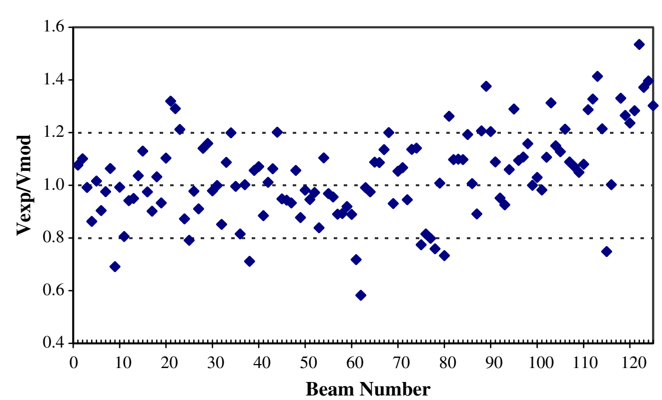

Fig. 6. Beams strengthened to flexure: comparison between experimental/analytical results.

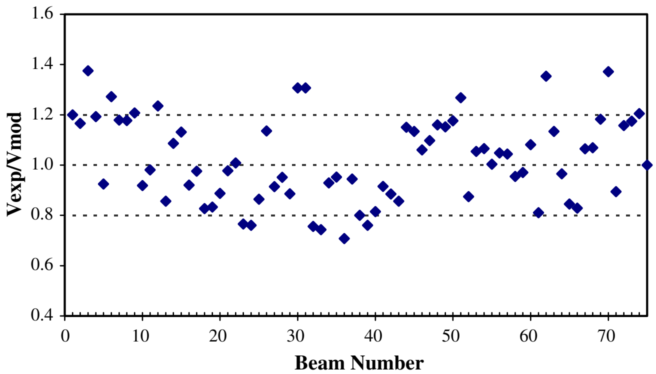

Fig. 7. Beams strengthened to shear: comparison between experimental/analytical results.

workmanship also plays an important role in the lack of consistency on the reported mode of failures.

6.2. RC beams strengthened for shear

Fig. 7 shows the comparison between experimental results and the analytical predictions based on the proposed model. As in Fig. 6, V_{mod} is the minimum of all calculated failure loads. The results again confirm that the failure loads predicted by the proposed model agree very well with the test results. The mean $V_{\mathrm{exp}}/V_{\mathrm{mod}}$ is 1.02, with a COV of 16.4%. The predicted failure modes (not reported here) also show a generally good correlation with the observed failure modes.

7. Conclusions

In this paper a general philosophy for the design and analysis of plate-bonded RC beams is provided. In particular, to address critical issues in the process of achieving sustainable development in the construction industry, the need of a global strategy is discussed. Such an approach, involves choice of materials, design, construction, maintenance, repair and rehabilitation, integrating material properties with structural performance.

The application of the proposed concepts is not only validated by an extensive multiphase experimental research carried out by the authors but also against the results of more than 200 tests of strengthened beam to flexure and/or shear reported in the literature.

In fact, the main results of an experimental/analytical research program, developed by the authors in recent years, are summarized and critically analyzed, in order to highlight the most relevant mechanical and analytical parameters which influence the structural behaviour of plate-bonded RC members under bending and shear.

Then, as an example of the application of a holistic approach, a unified global model is presented to predict failure loads and modes of RC beams strengthened to flexure and/or shear by externally bonded plates. The model is capable of evaluating the hierarchy of resistance of the internal ring chain in a strengthened RC beam. As a result, it can be an effective tool for developing design procedures capable of ensuring safe and ductile performances of strengthened RC beams.

The effectiveness of this design procedure can be assured by predicting the internal strengths corresponding to the different failure modes of the member. Thus, it is possible to select geometry and mechanical properties of the composite material to ensure fewer unfavourable failure modes.

The proposed design strategy allows for the evaluation of the internal strengths of the bonded member based on a *unified* approach which gives a good correlation with available test data and represents a valid holistic design tool which matches material characteristics with the required structural integrity and ductility.

References

[1] Al-Sulaimani GJ, Sharif A, Basunbul IA, Baluch MH, Ghaleb BN. Shear repair for reinforced concrete by fiberglass plate bonding. ACI Struct J 1994;91(4):458–64.

[2] An W, Saadatmanesh H, Ehsani MR. RC beams strengthened with FRP plates. II: Analysis and parametric study. J Struct Eng, ASCE 1991;117(11):3434–55.

[3] Ascione L, Feo L. Modeling of composite/concrete interface of RC beams strengthened with composite laminates. Composites: Part B, vol. 31. Elsevier; 2000. p. 535–40.

[4] Bencardino F, Colotti V, Spadea G, Swamy RN. Failure analysis of RC beams with bonded FRP laminates. In: Figueiras J et al., editors. In: Proc Int Conf Composites in Constructions—CCC2001, Porto, Portugal, 10–12 October, vol. 1, 2001. p 505–10.

[5] Bencardino F, Spadea G, Swamy RN. The role of shear in RC beams strengthened with CFRP laminates. In: Proc Int Conf Composites in Constructions—CCC2003, Rende, Italy, 16–19 September, 2003. p. 409–14.

[6] Bencardino F. Numerical analysis and experimental comparisons of RC beams externally strengthened by CFRP. iiC—l'Industria Italiana del Cemento, January, n. 794, 2004. p. 60–82.

[7] Bencardino F, Spadea G, Swamy RN. RC beams strengthened with CFRP laminates subjected to shear loading regime. In: La Tegola, A, Nanni A, editors. In: Proc 1st Int Conf, on IMTCR-04, Lecce, Italy, 6–9 June 2004. 2004;2:205–20.

[8] Bencardino F, Colotti V, Spadea G, Swamy RN. Shear behavior of reinforced concrete beams strengthened in flexure with bonded carbon fibre reinforced polymers laminates. Can J Civil Eng 2005;32(5):812–24.

[9] Buyle-Bodin F, David E, Ragneau E. Finite element modelling of flexural behaviour of externally bonded CFRP reinforced concrete structures. Engineering structures, Elsevier 2002;24(11):1423–29.

[10] Colotti V, Spadea G. Shear strength of RC beams strengthened with bonded steel or FRP plates. J Struct Eng, ASCE 2001;127(4):367–73.

[11] Colotti V, Spadea G, Swamy RN. Analytical model to evaluate failure behaviour of plated reinforced concrete beams strengthened for shear. ACI Struct J 2004;101(6):755–64.

[12] Khalifa A, Nanni A. Improving shear capacity of existing RC T-section beams using CFRP composites. Cement Concrete Compos 2000;22(3):165–74.

[13] Kishi N, Zhang G, Mikami H. Numerical cracking and debonding analysis of RC beams reinforced with FRP sheet. J Compos Construct, ASCE 2005;9(6):507–14.

[14] Jones R, Swamy RN, Charif A. Plate separation and anchorage of reinforced concrete beams strengthened by epoxy-bonded steel plates. Struct Engineer 1988;66(5):85–94.

[15] Mays GC, Peh LC. Strengthening of reinforced concrete beams in shear. In: Forde, M.C. editor. In: Proc Int Conf Structural Faults Repair. 1989;1:139–48.

[16] Mehta PK. High performance, high-volume fly ash concrete for sustainable development. Workshop on sustainable development and concrete technology. Beijing, China: CTRE Iowa State University; 2004. p. 3–14.

[17] Meier U, Kaìser H. Strengthening of structures with CFRP laminates. Advanced composite materials in civil engineering structures. Am Soc of Civil Engineers; 1991.

[18] Meier U, Deuring M, Meier H, Schwegler G. Strengthening of structures with CFRP laminates: research and applications in Switzerland. In: Neele KW, Labossiere P, editors. Advanced composite materials in bridges and structures. Canadian Soc of Civ Engineers; 1992. p. 243–51.

[19] Mukhopadhyaya P, Swamy RN. Interface shear stress: a new design criterion for plate debonding. J Compos Construct, ASCE 2001;5(1): 35–43.

[20] Nielsen MP. Limit analysis and concrete plasticity. Englewood Cliffs, NJ: Prentice-Hall; 1984.

[21] Ritchie PA, Thomas DA, Lu LW, Connelly GM. External reinforcement of concrete beams using fiber-reinforced plastics. ACI Struct J 1991;88(4):490–500.

[22] Saadatmanesh H, Ehsani MR. Fiber composite plates can strengthen beams. Concr Int 1990;12(3):65–71.

[23] Saadatmanesh H, Ehsani MR. RC beams strengthened with GFRP plates: I Experimental study. J Struct Eng, ASCE 1991;117(11): 3417–33.

[24] Sharif A, Al-Sulaimani GJ, Basunbul IA, Baluch MH, Ghaleb BN. Strengthening of initially loaded reinforced concrete beams using FRP plates. ACI Struct J 1994;91(2):160–8.

[25] Spadea G, Bencardino F, Swamy RN. Optimizing the performance characteristics of beams strengthened with bonded CFRP laminates. RILEM Mater Struct 2000;33:119–26.

[26] Spadea G, Swamy RN, Bencardino F. Strength and ductility of RC beams repaired with bonded CFRP laminates. J Bridge Eng ASCE 2001;6(5):349–55.

[27] Swamy RN, Jones R, Charif A. Contribution of externally bonded steel plate reinforcement to the shear resistance of reinforced concrete beams. In: Swamy RN, Gaul R, editors. Proc repair and strengthening of concrete members with adhesive bonded plates, vol. SP-165. Detroit: American Concrete Institute; 1996. p. 1–23.

[28] Swamy RN, Mukhopadhyaya P, Lynsdale CJ. Strengthening for shear of RC beams by external plate bonding. Struct Engineer 1999;77(12):19–30.

[29] Swamy RN. Designing concrete and concrete structures for sustainable development. Proc CANMET/ACI Int Symp on concrete technology for sustainable development. Canada: Vancouver; 2000.

[30] Swamy RN. Holistic design: key to sustainability in concrete construction. Struct Build 2001;146(4):371–9.

[31] Swamy RN. High performance cement based materials holistic design for sustainability in construction. In: Proc AMAS course on high performance cement based materials HPCBM'03. Warsaw, Poland, October 16–18, 2003. p. 37–81.

[32] Teng JG, Zhang JW, Smith ST. Interfacial stresses in reinforced concrete beams bonded with a soffit plate: a finite element study. Constr Build Mater 2002;16(1):1–14.

[33] Triantafillou TC, Plevris N. Strengthening of RC beams with epoxy-bonded fibre-composite materials. Mater Struct 1992;25:201–11.

[34] Triantafillou TC. Shear strengthening of reinforced concrete beams using epoxy-bonded FRP composites. ACI Struct J 1998;95(2): 107–15.

[35] Wong RSY, Vecchio FJ. Towards modeling of reinforced concrete members with externally bonded fiber-reinforced polymer composites. ACI Struct J 2003;100(1):47–55.

[36] Yang ZJ, Chen JF, Proverbs D. Finite element modelling of concrete cover separation failure in FRP plated RC beams. Constr Build Mater 2003;17(1):3–13.